and the tension side remains in tension. When the excitation is between the two critical speeds, the shaft undergoes two reversals in stress per revolution.

The balancing of machines and field balancing' are further examples of critical speed calculations. Since the subject is usually covered in the dynamics of machines, it will not be pursued here.

Case 3. Vibration Isolation and Transmissibility

Machines are often mounted on springs and dampers as shown in Fig. 3-23 to minimize the transmission of forces between the machine m and its foundation.

We shall first consider the system illustrated in Fig. 3-23(*a*). If a harmonic force is applied to m and the deflection of the foundation is negligible, the equation of motion is identical to Eq. (3-22). The force transmitted to the foundation is the sum of the spring force kx and the damping force $c\dot{x}$.

$$\text{Force transmitted} = kx + c\dot{x}$$

If the excitation is harmonic, the magnitude and the phase angle of the excitation force F_{eq} and the other forces are as illustrated in Fig. 3-24. The phase angle γ is generally is of secondary interest. Using Eq. (3-24), the force transmitted $\bar{F}_T$ is

$$\bar{F}_T = k\bar{X} + j\omega c\bar{X} = \frac{k + j\omega c}{k - \omega^2 m + j\omega c}\bar{F}_{eq} \tag{3-31}$$

The ratio of the amplitude of the force transmitted F_T and the amplitude of the driving force F_{eq} is called the ***transmissibility*** TR. From the equation above, we have

$$\text{TR} = \frac{\sqrt{1 + (2\zeta r)^2}}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}} \tag{3-32}$$

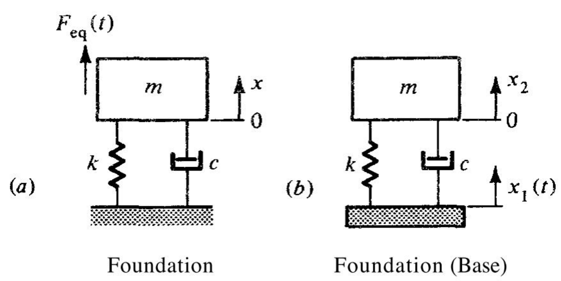

FIG. **3-23.** ***Vibration isolation.***

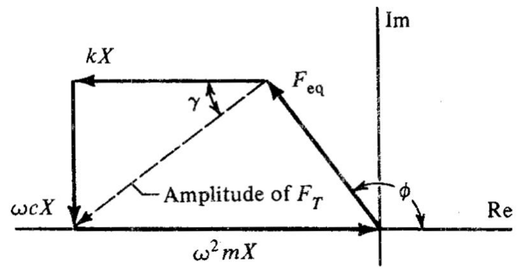

FIG. 3-24. Relation of force transmitted and other force vectors.

where $r = \omega/\omega_n$ and $c\omega/k = 2\zeta r$. The equation is plotted in Fig. 3-25. Note that all the curves in the figure cross at $r = \sqrt{2}$. Hence the transmitted force is greater than the driving force below this frequency ratio and less than the driving force when the machine is operated above this frequency ratio.

For a constant speed machine, the amplitude of the exciting force F_{eq} is constant. Hence the force transmitted is proportional to the value of the

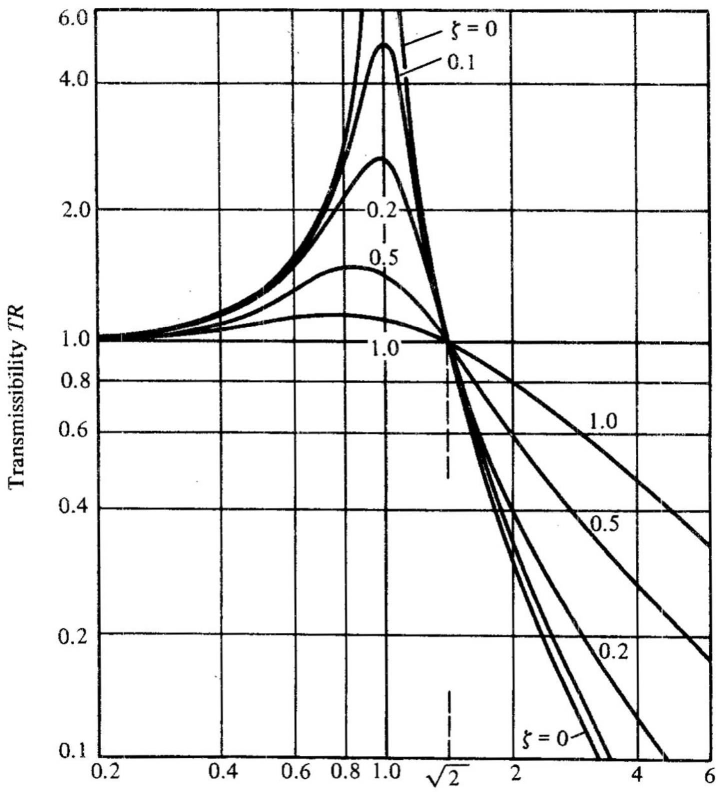

FIG. 3-25,. Transmissibility versus frequency ratio; system shown in Fig. 3-23(a).

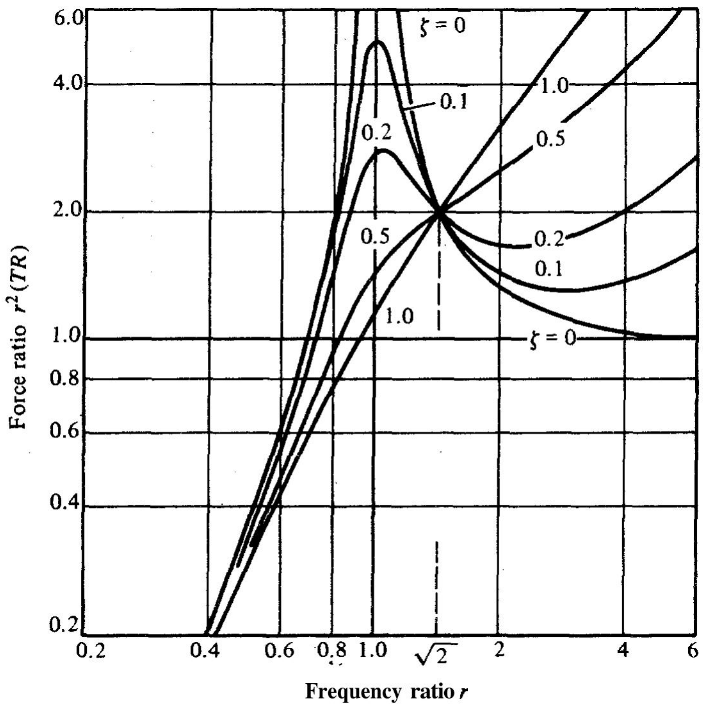

FIG. 3-26. Force ratio versus frequency ratio for inertial excitation; system shown in Fig. 3-15.

transmissibility TR. It is advantageous to operate a constant speed machine at $\omega > \sqrt{2}\omega_n$.

For a variable speed machine, the driving force F_{eq}, due to an unbalance me, is $me\omega^2$, where ω is the operating frequency. Let us define a constant force $F_n = me\omega_n^2$. Substituting $F_{eq} = me\omega^2$ into Eq. (3-31), dividing both sides of the equation by F_n, and simplifying, we obtain

$$\frac{F_T}{F_n} = \frac{r^2\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} = r^2(\mathrm{TR}) \tag{3-33}$$

where TR is as defined in Eq. (3-32). Hence the magnitude of the force transmitted can be high in spite of the low transmissibility. The equation is plotted in Fig. 3-26.

The reduction of the force transmitted in buildings is of interest. For example, the mechanical equipment of a tall office building is often located on the roof directly above the penthouse or the boardroom of the company.

The fractional reduction of the force transmitted is

$$\text{Force reduction} = \frac{F_{eq} - F_T}{F_{eq}} = 1 - \mathrm{TR}$$

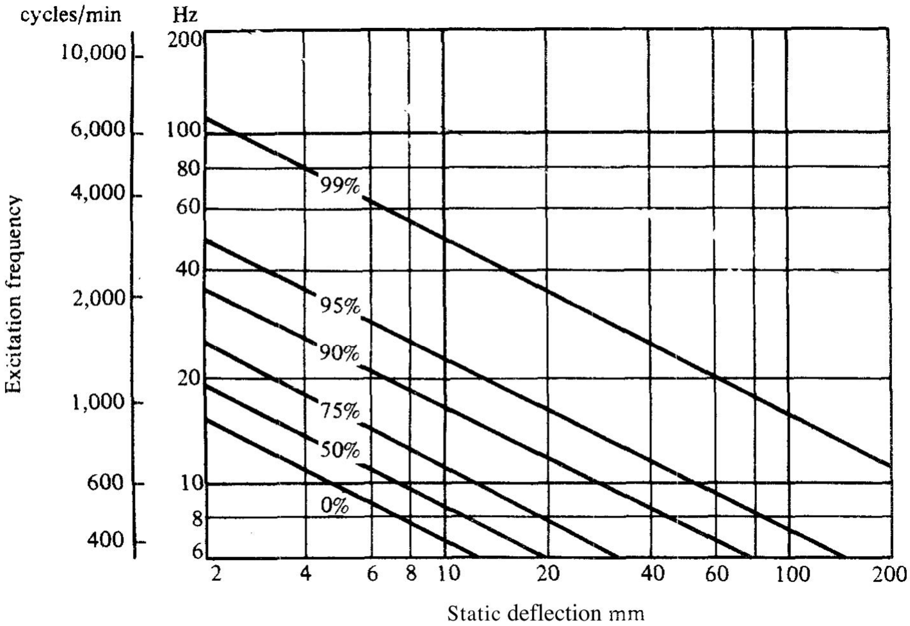

FIG. 3-27. Percentage reduction in force transmitted to foundation in *vibration* isolation, $\zeta = 0$.

where F_{eq} and F_T are the amplitudes of the excitation and the transmitted force, respectively. It is observed in Fig. 3-25 that low natural frequency and low damping are desirable for vibration isolation. Assume $\zeta \simeq 0$ and $r > 1$ in Eq. (3-32). Thus, $TR = 1/(r^2 - 1)$ and the force reduction becomes

$$\text{Force reduction} = \frac{r^2 - 2}{r^2 - 1}$$

Since $r^2 = (\omega/\omega_n)^2$, $\omega_n^2 = k/m$, and the static deflection of a spring $\delta_{st} = mg/k$, the equation above reduces to

$$\text{Force reduction} = \frac{\omega^2 \delta_{st} - 2g}{\omega^2 \delta_{st} - g} \tag{3-34}$$

The equation is plotted in Fig. 3-27

Example 17

An air compressor of 450 kg mass (992 lb,) operates at a constant speed of 1,750 rpm. The rotating parts are well balanced. The reciprocating parts are of 10 kg (22 lb,). The crank radius is 100 mm (4 in.). If the damper for the mounting introduces a damping factor $\zeta = 0.15$, (a) specify the springs for the mounting such that only 20 percent of the unbalance force is transmitted to the foundation, and (b) determine the amplitude of the transmitted force.