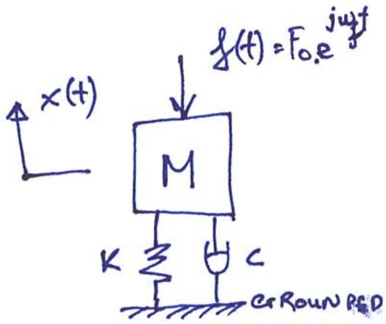
Assuming the solutions for FORCED VIBRATING SYSTEM at initial condition:

VIBRATION INCLUDED : FREE VIBRATION + FORCED VIBRATION

$x(t) = x_n(t) + x_f(t)$

→ As called : "homogeneous" + "particulars"



"homogeneous": $t \to \infty$ → "0" DISAPPEARED

"particulars": Steady state * → Resolving.

AMARINE

* Assume the harmonic motion : $F(t) = F_0 . e^{j\omega_f t}$ → then $x(t) = X_0 . e^{j\omega_f t}$ → $\begin{cases} \dot{x}(t) = j\omega_f X_0 . e^{j\omega_f t} \\ \ddot{x}(t) = -\omega_f^2 X_0 . e^{j\omega_f t} \end{cases}$

EOM : $m.\ddot{x}(t) + c.\dot{x}(t) + K.x(t) = F(t)$

$\Leftrightarrow \; -\omega_f^2 . m . e^{j\omega_f t} + j.\omega_f . c . X_0 e^{j\omega_f t} + K . X_0 e^{j\omega_f t} = F_0 . e^{j\omega_f t}$

$\Rightarrow X_0 = \dfrac{F_0}{-\omega_f^2 m + j\omega_f c + K}$ *

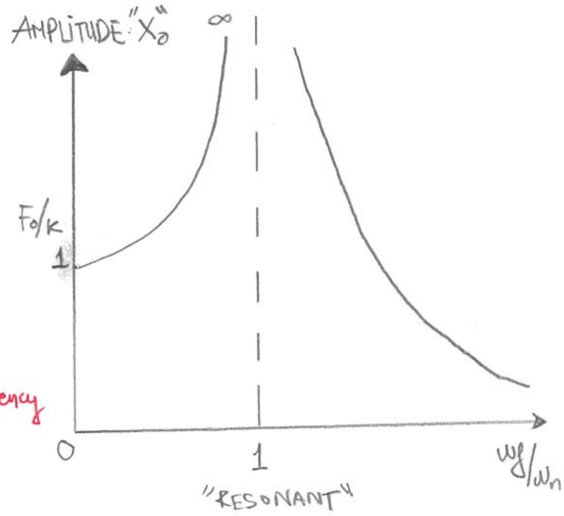
CASE I : UNDAMPED (C=0)

$X_0 = \dfrac{F_0}{-\omega_f^2 . m + K} = \dfrac{F_0}{\omega_n^2 - \omega_f^2}$

ω_n ↓ Natural frequency

ω_f ↓ Force frequency

$\Rightarrow \boxed{X_0 = \dfrac{F_0}{K} \cdot \dfrac{1}{1 - \left(\dfrac{\omega_f}{\omega_n}\right)^2}}$



CASE II : DAMPED (C≠0)

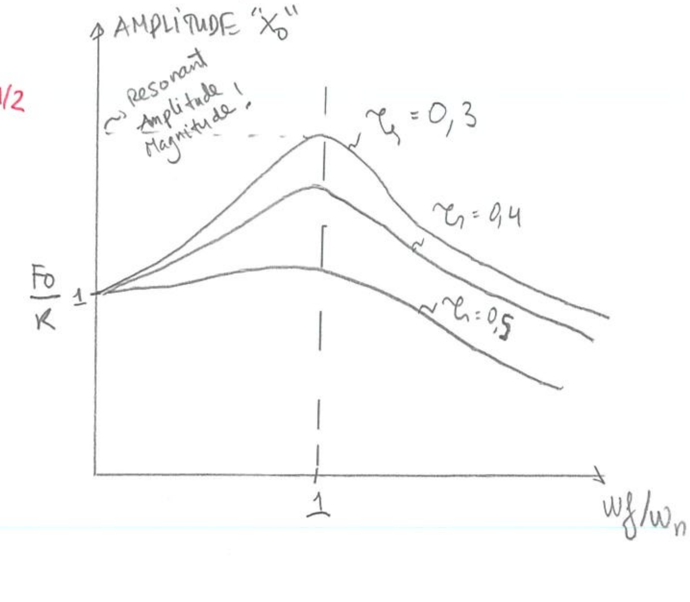
$\Rightarrow \left| \dfrac{X_0 . K}{F_0} \right| = \dfrac{1}{\left[\left[1 - \left(\dfrac{\omega_f}{\omega_n}\right)^2\right]^2 + \left(2.\zeta . \dfrac{\omega_f}{\omega_n}\right)^2 \right]^{1/2}} . e^{-i\varphi}$

* $\zeta = \dfrac{c}{c_c}$ → see FREE NATURAL DAMPED case.

Note: if we use the $F(t) = F_0 \sin \omega_f t$, the result is more easy to understand :

$\Rightarrow \dfrac{X_0}{F_0} K = \dfrac{1}{\left[\left[1 - \left(\dfrac{\omega_f}{\omega_n}\right)^2\right]^2 + \left(2\zeta \dfrac{\omega_f}{\omega_n}\right)^2 \right]^{1/2}}$

ζ ↓ Damping ratio



Support: $a + bi = A . e^{i\varphi}$

* $A = \sqrt{a^2 + b^2}$

* $\varphi = tg \dfrac{b}{a}$