CHEMICAL ENGINEERING

FACTS AT YOUR FINGERTIPS

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HEAT TRANSFER

BASIC EQUATIONS FOR HEAT TRANSFER

Conduction:

$$\frac{q}{A} = k\left(\frac{dT}{dx}\right) \tag{1}$$

Convection:

$$q = h_{avg}A_s\left(T_s - T_f\right) \tag{2}$$

Radiation

$$\frac{q}{A} = \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right) \tag{3}$$

HEAT TRANSFER THROUGH SHELL-AND-TUBE HEAT EXCHANGERS

Heat duty for exchanger transferring sensible heat:

$$q = \dot{m}C_{p,avg}\left(T_o - T_i\right) \tag{4}$$

For use in heat-exchanger calculations, Equation (2) above is often written as follows:

$$q = UA_s\Delta T \tag{5}$$

where U can be calculated from the following relationship;

$$\frac{1}{U} = \frac{1}{h_o} + \frac{1}{h_i\left(D_i / D_o\right)} + \frac{1}{h_w} + \frac{1}{h_s} \tag{6}$$

Various equations are available (see the references) for calculating h_i and h_o, depending on such factors as the Reynolds numbers for the flowing fluids and whether the fluids undergo sensible heat transfer or, instead, vaporization or condensation. For instance, for sensible heat transfer with fluids under forced convection in fully turbulent flow inside tubes with sharp-edged entrances, the following, well established relationship involving the Nusselt, Reynolds and Prandtl numbers holds:

$$\frac{h_iD_i}{k} = 0.023\left(\frac{D_i\dot{m}}{\mu}\right)^{0.8}\left(\frac{c_p\mu}{k}\right)^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14} \tag{7}$$

With the assumption that the (μ/μ_w) term can be ignored, the immediately above equation has been rearranged as follows [*1*, *2*] to facilitate assessing the effects of fluid (and system) properties upon h_i (assuming sensible heat transfer, full turbulence, fluid inside tubes):

$$h_i = 0.023\,\frac{\dot{m}^{0.8}k^{2/3}c_p^{\;1/3}}{D_i^{\;0.2}\mu^{0.47}} \tag{8}$$

For sensible heat transfer with fluids under forced convection flowing across tube banks (thus, outside the tubes), the following relationship has been published [*3*]

$$\frac{h}{c\dot{m}} = \frac{a}{\left({}^{c\mu}\!/\!_{k}\right)^{2/3}\left({}^{D_o\dot{m}}\!/\!_{\mu}\right)^{m}\left({}^{\mu_w}\!/\!_{\mu}\right)^{0.14}} \tag{9}$$

In this equation, the values of a and m are to be as follows:

Tube pattern	Reynolds number	m	a
Staggered	above 200,000	0.300	0.166
Staggered	300 to 200,000	0.365	0.273
Staggered	less than 300	0.640	1.309
Inline	above 200,000	0.300	0.124
Inline	300 to 200,000	0.349	0.211
Inline	less than 300	0.569	0.742

For an excellent discussion of the Equation (6) fouling factor, h_s, see Reference [*4*].

The appropriate ΔT depends on the configuration of the heat exchanger (see references). For example, for a simple countercurrent-flow exchanger, the appropriate temperature (referred to as the log mean temperature difference, ΔT_{LM}), is found as follows;

$$\Delta T_{LM} = \frac{\left(T_h - t_h\right) - \left(T_c - t_c\right)}{\ln\left(T_h - t_h\right) / \left(T_c - t_c\right)} \tag{10}$$

Energy balance for a heat exchanger

If any heat exchange with the ambient air is neglected, the following relationship is valid;

$$\dot{m}t_h\left(H_{ha} - H_{hb}\right) = \dot{m}t_c\left(H_{cb} - H_{ca}\right) = q \tag{11}$$

BATCH HEATING

For heating a batch of fluid from temperature T_1 to T_2, by means of an internal coil of area A and an isothermal heating medium at temperature T, the following relationship holds:

$$\ln\left(\frac{T - T_1}{T - T_2}\right) = \left(\frac{UA}{cM}\right)\theta \tag{12}$$

STEADY-STATE HEAT FLOW BY CONDUCTION

For conduction through a homogeneous plane wall of thickness x and constant (or average) thermal conductivity k,

$$\frac{q}{A} = k\,\frac{\Delta T}{x} \tag{13}$$

where ΔT is the temperature difference through the wall

For conduction through a three-layer plane wall (for example, a wall with thermal insulation on each side), having layers of thicknesses x_1, x_2 and x_3 and thermal conductivities k_1, k_2 and k_3,

$$q = \frac{\Delta T}{{}^{x_1}\!/\!_{k_1A} + {}^{x_2}\!/\!_{k_2A} + {}^{x_3}\!/\!_{k_3A}} \tag{14}$$

where ΔT is the overall temperature difference across all three layers

For conduction through the wall of a cylinder of length L, whose inner and outer radii are r_{inner} and r_{outer}, with inner and outer walls at temperatures $T_{s,inner}$ and $T_{s,outer}$,

$$q = \frac{k(2\pi L)(T_{s,inner} - T_{s,outer})}{\ln\left({}^{r_{outer}}\!/\!_{r_{inner}}\right)} \tag{15}$$

References

1. Guffey, G.E., Sizing Up Heat Transfer Fluids and Heaters, *Chem. Eng.*, pp. 126–131, October 1997.
2. McCabe, Smith and Harriott, "Unit Operations of Chemical Engineering," McGraw-Hill.
3. Chopey, ed., "Handbook of Chemical Engineering Calculations,", McGraw-Hill.
4. Polley, Put Fouling in Its Place, *Chem. Eng.*, pp. 46-49, December 2002.
5. Incropera and DeWitt, "Introduction to Heat Transfer," Wiley.
6. Jones, Thermal Design of the Shell-and-Tube, *Chem. Eng.*, pp. 60–65, March 2002.

NOMENCLATURE:

A	cross-sectional area perpendicular to the flow of heat
a	parameter in convection-coefficient equation
A_s	surface area
c, C_p	specific heat; specific heat at constant pressure
$C_{p,\,avg}$	specific heat at average fluid temperature
D_i	inner diameter of heat-exchanger tube
D_o	outer diameter of heat-exchanger tube
H_{ca}, H_{ha}	enthalpy per unit mass of entering cold and warm fluid, respectively
H_{cb}, H_{hb}	enthalpy per unit mass of exiting cold and hot fluid, respectively
h_{avg}	average convection coefficient
h_i	convection coefficient for inner tube wall
h_o	convection coefficient for outer tube wall
h_s	fouling heat-transfer coefficient
h_w	coefficient of heat-transfer radially through tube wall; a function of tube thickness and thermal conductivity
k	thermal conductivity
L	length
M	weight of batch
m	parameter in convection-coefficient equation
$\dot{m}$	mass flowrate of fluid
$\dot{m}\,t_c$	mass flowrate of cold fluid
$\dot{m}\,t_h$	mass flowrate of hot fluid
q	rate of heat flow
T	temperature (for radiation calculations, use absolute temperature)
T_c	in a heat exchanger, the exit temperature for the stream being cooled
T_f	temperature of fluid
T_h	in a heat exchanger, the inlet temperature for the stream being cooled
T_i	inlet temperature
T_o	outlet temperature
T_s	temperature of surface (for radiation calculations, use absolute temperature)
T_{sur}	temperature of surroundings (for radiation calculations, use absolute temperature)
t_c	in a heat exchanger the inlet temperature for the stream being heated
t_h	in a heat exchanger, the outlet temperature for the stream being heated
ΔT_{LM}	log mean temperature difference
$\Delta T/dx$	temperature gradient during conductive heat flow
U	overall heat transfer coefficient
x	distance the heat flows during conduction
ε	emissivity
μ	viscosity; viscosity at bulk fluid temperature
μ_w	viscosity at tube-wall temperature
σ	Stefan-Boltzmann constant
θ	time required for batch heating